THE COMPLETE CLASSIFICATION OF UNITAL GRAPH C^*-ALGEBRAS: GEOMETRIC AND STRONG

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Abstract

We provide a complete classification of the class of unital graph C^*-algebras—prominently containing the full family of Cuntz–Krieger algebras—showing that Morita equivalence in this case is determined by ordered, filtered K-theory. The classification result is geometric *in the sense that it establishes that any Morita equivalence between $C^*(E)$ and $C^*(F)$ in this class can be realized by a sequence of moves leading from E to F, in a way resembling the role of Reidemeister moves on knots. As a key ingredient, we introduce a new class of such moves, we establish that they leave the graph algebras invariant, and we prove that after this augmentation, the list of moves becomes complete in the sense described above. Along the way, we prove that every (reduced, filtered) K-theory order isomorphism can be lifted to an isomorphism between the stabilized C^*-algebras—and, as a consequence, that every such order isomorphism preserving the class of the unit comes from a $*$-isomorphism between the unital graph C^*-algebras themselves. It follows that the question of Morita equivalence and $*$-isomorphism among unital graph C^*-algebras is a decidable one. As immediate examples of applications of our results, we revisit the classification problem for quantum lens spaces and we verify, in the unital case, the Abrams–Tomforde conjectures.*

Contents

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